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Little string theory in a double scaling limit

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Abstract: A double scaling limit can be defined in string theory on a Calabi-Yau (CY) manifold by approaching a point in moduli space where the CY space develops an isolated singularity and at the same time taking the string coupling to zero, while keeping a particular combination of the two parameters fixed. This leads to a decoupled theory without gravity which has a weak coupling expansion, and can be studied using a holographically dual non-critical superstring description. The usual "Little String Theory" corresponds to the strong coupling limit of this theory. We use holography to compute two and three point functions in weakly coupled double scaled little string theory, and study the spectrum of the theory in various dimensions. We find a discrete spectrum of masses which exhibits Hagedorn growth.

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Contents

1. The double scaling limit

In the last few years it became clear that consistency of string theory implies the existence of new non-local theories without gravity in six or less dimensions. One way of defining these "little string theories" (LST's) is to consider string dynamics in vacua which contain $NS5$-branes, in the decoupling limit $g_s \to 0$ [1]. In this limit, fluctuations in the bulk of spacetime decouple, and one is left with the dynamics of modes which live on the fivebranes. For example, taking this limit in type IIA or IIB string theory in flat spacetime with N fivebranes leads to maximally supersymmetric six dimensional theories (sixteen supercharges) with $(2, 0)$ or $(1, 1)$ supersymmetry, respectively.

LST's do not contain gravity but have other features that are reminiscent of critical string theories. In particular, they have a Hagedorn density of states, and upon compactification on tori they exhibit T-duality. There are some apparent differences as well:

1. Unlike critical strings, for which it is believed that one can only study on-shell physics (e.g. the S-matrix), LST's are expected to have well defined off-shell Green's functions.

2. Critical string theory in flat non-compact spacetime has a fundamental scale, the string scale l_s, and a dimensionless coupling, g_s. For small g_s, the theory is weakly coupled. LST's, as defined above, have a scale l_s, and are further labeled by the number of fivebranes N. They appear to be inherently strongly

coupled. One might be tempted to associate $1/N$ with a coupling in LST, but there is no known sense in which the theory becomes weakly coupled at large N (it does become weakly coupled at large N for sufficiently low energies). Nevertheless, we will see that one can in fact define LST's which are arbitrarily weakly coupled, so in this respect LST is actually similar to critical string theory.

Another definition of LST, which will be more useful for our purposes, involves the study of string vacua of the form $\mathbb{R}^{d-1,1} \times \mathcal{M}$, where $\mathcal{M}$ is a CY manifold with an isolated singularity, in the decoupling limit $g_s \to 0$. Since in this limit all non-trivial physics is localized near the singularity, to study the resulting LST one can replace the CY manifold $\mathcal{M}$ by its form near the singular point.

In the maximally supersymmetric, six dimensional case of [1], $\mathcal{M} = K3$ and the possible singularities have an ADE classification. The theory on N $NS5$-branes mentioned above corresponds to an A_{N-1} singularity, near which the $K3$ manifold looks like an ALE space — the singular, non-compact hypersurface

$$z_1^N + z_2^2 + z_3^2 = 0 \tag{1.1}$$

in $\mathbb{C}^3$. The two descriptions of six dimensional LST given above are related by T-duality [2, 3]. The theory on N type IIA $NS5$-branes is equivalent to the IIB theory on $\mathbb{R}^{5,1}$ times the ALE space (1.1), and vice versa.

An interesting generalization of the above procedure is the following. Let t be one of the moduli of $K3$ CFT, and $t = t_c$ be the value of t for which the $K3$ surface develops an isolated singularity. The standard LST limit is $t \to t_c$, followed by $g_s \to 0$. Instead, one can consider a double scaling limit, i.e. take $t \to t_c$ and $g_s \to 0$ at the same time, keeping some combination, $(t - t_c)^\gamma / g_s$, fixed.

To describe this limit more precisely it is convenient to blow up the vicinity of the singularity and replace the $K3$ surface by the ALE space (1.1) describing the region near the singularity (at $z_a = 0$). One way of smoothing the singularity is to replace (1.1) by

$$z_1^N + z_2^2 + z_3^2 = \mu \,, \tag{1.2}$$

where μ can be thought of as one of the moduli of the underlying $K3$ CFT ($\mu \sim t - t_c$ in the previous notation). As we will see later, string theory on $\mathbb{R}^{5,1}$ times the surface (1.2) depends on μ, g_s only through the scaling parameter $x \equiv \mu^{\frac{1}{N}}/g_s$. The double scaling limit corresponds to $\mu \to 0$, $g_s \to 0$ with x held fixed. Note that:

1. The string coupling expansion corresponds in this case to an expansion in powers of $1/x$.

2. The original definition of LST corresponds to $x = 0$ ($\mu \to 0$ first, then $g_s \to 0$), and hence gives rise to a strongly coupled theory.

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3. While in string theory on $K3$ one actually has to take the double scaling limit to decouple the LST from gravity and bulk string theory, replacing the $K3$ surface by (1.2) automatically implements the limit. In particular, μ does not have to be small in (1.2); in fact, the LST becomes weakly coupled at *large* μ.

4. As emphasized in [4], the double scaling limit defined above is qualitatively very similar to the one used in the study of two dimensional critical string theory [5], and one can borrow much of the intuition developed there.

5. String theory on resolved ALE spaces was considered in some very interesting pre-holography papers [6, 2]. We will comment later on the relation of these papers to our work.

The deformation (1.2) has a natural interpretation in terms of the theory of N $NS5$-branes.[1] Rewriting (1.2) as

$$\prod_{j=1}^{N}\left(z_1 - r_0 e^{\frac{2\pi i j}{N}}\right) + z_2^2 + z_3^2 = 0\,, \qquad \mu = (-r_0)^N \tag{1.3}$$

and performing the T-duality of [2, 3], we find a system of N fivebranes distributed uniformly on a circle of radius r_0, which is related to μ as in (1.3). In type IIB string theory, the low energy theory on N $NS5$-branes is SU(N) SYM theory with sixteen supercharges, and (1.3) corresponds to a point in the moduli space of vacua of the gauge theory, in which a complex scalar field Φ in the adjoint representation of the gauge group has a v.e.v.

$$\langle\Phi\rangle = C\,\mathrm{diag}\left(e^{\frac{2\pi i}{N}}, e^{\frac{4\pi i}{N}}, \ldots, e^{\frac{2\pi i N}{N}}\right). \tag{1.4}$$

The double scaling limit $\mu, g_s \to 0$ with $x = \mu^{1/N}/g_s$ fixed has a nice interpretation in this language. At the point (1.4) in moduli space, the SU(N) gauge symmetry is broken to U$(1)^{N-1}$, and the mass of the off-diagonal gluons ("W-bosons") is $m_W \sim r_0/g_s l_s \sim x/l_s$ (since W-bosons correspond to $D1$-branes stretched between the fivebranes).

Thus, the double scaling limit is simply the decoupling limit of [1] with fixed W-boson masses. The scaling parameter x is the scale of these masses (in string units). The double scaled theory is weakly coupled for energies well below m_W. For $E \gg m_W$ one expects to recover the strongly coupled LST at the origin [1]. A similar story (with slightly different details) can be told for type IIA fivebranes.

The above discussion can be generalized to higher dimensional Calabi-Yau spaces. One again studies the limit where the CY manifold develops an isolated singularity and blows up the region near the singularity to define the double scaling limit.

[1]We thank O. Aharony and T. Banks for discussions of this issue.

There are many possible singularities that can appear on CY manifolds. We will restrict our attention to quasi-homogenous hypersurface singularities, in which the vicinity of the singularity is (as in (1.1)) a hypersurface $F(z_1, \ldots, z_{n+1}) = 0$ in $\mathbb{C}^{n+1}$, where F is a quasi-homogenous polynomial with weight one under $z_a \to \lambda^{r_a} z_a$, i.e.

$$F(\lambda^{r_a} z_a) = \lambda F(z_a)\,, \qquad \lambda \in \mathbb{C}\,, \tag{1.5}$$

for some set of positive weights r_a. The fact that the singularity is isolated implies that F is transverse, i.e. the only point at which all derivatives $\partial_{z_a} F$ vanish is the origin, $z_a = 0$. The double scaling limit gives rise in this case to a $d = 10 - 2n$ dimensional non-gravitational theory. It involves string theory on $\mathbb{R}^{d-1,1}$ times the non-compact manifold

$$F(z_1, \ldots, z_{n+1}) = \mu\,. \tag{1.6}$$

We will see below that the combination of μ and g_s on which physics in this background depends is the scaling variable

$$x \equiv \frac{\mu^{r_\Omega}}{g_s}\,, \tag{1.7}$$

where

$$r_\Omega \equiv \sum_{a=1}^{n+1} r_a - 1\,. \tag{1.8}$$

We will restrict to $r_\Omega > 0$. The resulting LST is weakly coupled for large x and we will study it in this limit.

2. The holographic description

The construction described in the previous section is useful for establishing the existence of LST's, but it does not provide efficient techniques for studying them in detail. To go further, we will use a holographic description of these theories. This description, which is a generalization of the AdS/CFT correspondence [7], was proposed in [8] (see also [9, 10, 11, 12, 13, 14]) and was generalized to the case of general hypersurface singularities (1.6) in [4]. We will next briefly summarize it.

The claim is that holography relates the decoupled dynamics at the singular point $z_a = 0$ in string theory on $\mathbb{R}^{d-1,1}$ times the space $F(z_a) = 0$ (1.5) ("the boundary theory") to a "bulk theory," string theory on

$$\mathbb{R}^{d-1,1} \times \mathbb{R}_\phi \times S^1 \times LG(W = F)\,, \tag{2.1}$$

where $\mathbb{R}_\phi$ is the real line, labeled by ϕ, along which the dilaton varies linearly,

$$\Phi = -\frac{Q}{2}\phi\,, \tag{2.2}$$

and $LG(W = F)$ is a Landau-Ginzburg $N = 2$ SCFT of $n + 1$ chiral superfields z_a with superpotential $W(z_a) = F(z_a)$ (1.5). It is easy to check [4] that consistency of string propagation on (2.1) requires a relation between r_Ω (1.8) and Q (2.2):

$$Q^2 = 2r_\Omega\,. \tag{2.3}$$

We will label the S^1 in (2.1) by Y; its radius is determined by the GSO projection to be Q. The GSO projection further acts as an orbifold by a shift along the circle, $Y \to Y + \pi Q$, and (roughly) as fermion number on the rest of the theory (see [4] for the details).

For the six dimensional, maximally supersymmetric case (1.2), the superpotential is $W = z_1^N + z_2^2 + z_3^2$, and the LG model is an $N = 2$ minimal model, which can be thought of as the coset SCFT $\mathrm{SU}(2)_N/\,\mathrm{U}(1)$. The GSO projection acts as a Z_N orbifold on $\mathrm{SU}(2)_N/\,\mathrm{U}(1) \times S^1$, turning it into $\mathrm{SU}(2)_N$. The backgound (2.1) thus becomes $\mathbb{R}^{5,1} \times \mathbb{R}_\phi \times \mathrm{SU}(2)$, recovering the results of [15].

The theory described by (2.1) is singular. The string coupling $g_s \sim \exp(-Q\phi/2)$ blows up as $\phi \to -\infty$; the theory runs to strong coupling. This is not surprising in view of the earlier discussion, since (2.1) is dual to the singular background (1.6) with $\mu = 0$, or (1.7) $x = 0$, which as we mentioned is strongly coupled. One would like to turn on μ in (1.6) and study the dual theory for large x, where it is weakly coupled. Turning on μ in (1.6) corresponds [16, 4] in the dual theory (2.1) to adding to the worldsheet lagrangean the superpotential term

$$\delta\mathcal{L} = \mu \int d^2\theta e^{-\frac{1}{Q}(\phi+iY)} + \text{c.c.}\,. \tag{2.4}$$

Here ϕ, Y are worldsheet superfields whose bosonic components are the scalar fields ϕ, Y discussed above. A few comments about the interaction (2.4) are in order:

1. This is the $N = 2$ Liouville interaction. It preserves worldsheet $N = 2$ superconformal symmetry and hence spacetime supersymmetry. At the same time it prevents the dilaton from running to strong coupling.

2. Even before we add (2.4) to the lagrangean, the fact that this operator is in the spectrum of the theory implies that the radius of Y is an integer multiple of Q. An analysis of the rest of the spectrum shows [4] that this radius is equal to Q.

3. We can now derive (1.7). Standard scaling arguments (see e.g. [5]) imply that the perturbative expansion in the vacuum (2.1), (2.2) and (2.4) is a series in $g_s/\mu^{Q^2/2}$. Using (2.3), this leads to (1.7).

It is interesting to compare our discussion to the work of [6, 2]. In these papers it was pointed out that the sigma-model whose target space is the non-compact manifold (1.6) can be alternatively described by a Landau-Ginzburg model with superpotential

$$W = -\mu z_0^{-k} + F(z_1, \ldots, z_{n+1}), \tag{2.5}$$

where z_0 is an additional chiral superfield, and

$$k \equiv \frac{1}{r_\Omega}. \tag{2.6}$$

The description (2.5) is natural since the equation $W(z_0, z_a) = 0$ in $n+1$ dimensional weighted projective space satisfies the CY condition with $\hat{c} = n$, and if $z_0 \neq 0$ we can set it to one and recover the space (1.6). It was also argued in [6, 2] that the patch with $z_0 = 0$ should not appear since in the LG phase the superpotential (2.5) pushes z_0 to large values.

The first term in the superpotential (2.5) appears to be ill defined: the corresponding potential does not have a minimum at a finite value of z_0, and in general k is non-integer, which makes W non single valued. In [6, 2] it was proposed to interpret it as an SL(2)/ U(1) coset SCFT at level k, and arguments were presented to support this interpretation. Geometrically, this coset corresponds to a semi-infinite cigar [17], with the string coupling going to zero far from the tip and approaching an arbitrary finite value at the tip ($g_s \sim \mu^{-1/k}$).

Comparing (2.1), (2.4) with (2.5) we see that our description of (1.6) is very similar, but appears to be different from that of [6, 2]. For $\mu = 0$ the two are identical: both contain a LG model with superpotential $W = F$, and an infinite cylinder $\mathbb{R}_\phi \times S^1$ with a linear dilaton along $\mathbb{R}_\phi$. When μ is turned on, the two descriptions naively disagree. In our case, the strong coupling singularity at $\phi \to -\infty$ is cut off by a superpotential (2.4), whereas in [6, 2] it is eliminated by changing the topology of the cylinder to that of the semi-infinite cigar.

Since both descriptions are quite well motivated, we would like to propose that they are in fact equivalent, i.e. that $N = 2$ Liouville (with a cosmological constant μ) and SL(2)/ U(1) (with $e^\Phi \sim \mu^{-1/k}$ at the tip of the cigar) are isomorphic SCFT's. They are related by strong-weak coupling duality on the worldsheet (possibly, a kind of T-duality). We have not proven this statement but would like to offer the following comments in its support:

1. The matching of the central charges of the two theories relates the slope of the linear dilaton Q (2.2) to the level k of SL(2)/ U(1). The relation, which follows from (2.3), (2.6), is $Q^2 = 2/k$. Thus, if the two are equivalent, the relation between them is indeed a strong-weak coupling duality. $N = 2$ Liouville is weakly coupled for large Liouville central charge, i.e. large Q, while SL(2)/ U(1) is weakly coupled for large k (when the curvature of the cigar goes to zero everywhere).

2. Recall that the radius of the S^1 (labeled by Y) in $N = 2$ Liouville is Q. The asymptotic radius of the Euclidean cigar is $\sqrt{2k}$ [17]. The two are indeed inverses of each other, $R_Y = 2/R_{\text{cigar}}$, as appropriate for T-dual theories.

3. In $N = 2$ Liouville, the cosmological term breaks translation invariance in Y. In contrast, in $\mathrm{SL}(2)/\mathrm{U}(1)$ translation invariance around the cigar is not broken, but winding number is not a good symmetry, since winding modes around the cigar can move to the tip and contract to a point. This again is consistent with an interpretation of the relation between the two theories as T-duality, which exchanges winding and momentum modes. In both theories, as $\mu \to 0$ the respective symmetries are restored.

4. A closely related duality to the one advocated here appeared in unpublished work by V.A. Fateev, A.B. Zamolodchikov and Al.B. Zamolodchikov. It relates the bosonic $\mathrm{SL}(2)/\mathrm{U}(1)$ CFT to a bosonic analog of our $\mathbb{R}_\phi \times S^1$ with the superpotential (2.4) replaced by a bosonic potential of the form $e^{\beta\phi}\cos(\gamma Y)$.

Finally, we should remark on the possible relation of the duality proposed above to the standard T-duality with respect to the abelian isometry of the cigar CFT [18]. The latter takes the cigar into a trumpet, which seems different from the $N = 2$ Liouville background at small ϕ. However, it might be that non-perturbative worldsheet effects modify the trumpet background. The abelian T-duality of an ALE space, mentioned previously, is an example where the naive dual background is expected to receive worldsheet instanton corrections which modify it to the (near-horizon geometry of the) $NS5$-brane background (more generally, the abelian dual of a Kaluza-Klein monopole is expected to receive worldsheet instanton corrections which modify it to an H-monopole [19]).

3. Correlation functions and spectrum

In this section we will use the holographic description of [8, 4] to compute correlation functions in weakly coupled double scaled little string theory. Recall that, in general, holography relates observables in the non-gravitational "boundary theory" to non-normalizable on-shell states in the "bulk theory." A large class of such observables corresponds to non-normalizable vertex operators (i.e. fundamental string states) in the bulk theory, and we will focus on those here.

To get started, we need to decide which of the two T-dual descriptions of the bulk theory to use, the $\mathrm{SL}(2)/\mathrm{U}(1)$ or $N = 2$ Liouville one. As explained in section 2, the two are supposed to be equivalent, but each is more appropriate in a different region of parameter space. For large k it is better to use $\mathrm{SL}(2)/\mathrm{U}(1)$, while for small k the $N = 2$ Liouville description provides a better qualitative guide to the physics.

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The transition between the two descriptions occurs at $k=1$ (corresponding to the self dual radius $R_Y = R_{\rm cigar} = \sqrt{2}$).

In our context, k lies in the range[2] $1/2 \le k < \infty$. For example, in the maximally supersymmetric six dimensional case we have $k = N$, the number of fivebranes, and it can become arbitrarily large, but is bounded from below by two. Therefore, it is natural to use the SL(2)/ U(1) variables to describe the observables and compute correlation functions. We next recall a few facts about SL(2)/ U(1) CFT.

3.1 SL(2)/ U(1) CFT

A convenient description of the SL(2)/ U(1) coset CFT is obtained by starting with SL(2) $\times$ U(1) and gauging the null diagonal U(1) obtained from J^3 of SL(2) and the extra U(1) (and similarly for the other worldsheet chirality). Realizing the extra U(1) in terms of a canonically normalized free scalar field Y, we gauge the axial symmetry

$$\left(J^3 - i\sqrt{\frac{k}{2}}\partial Y, -\bar{J}^3 - i\sqrt{\frac{k}{2}}\bar\partial \bar{Y}\right), \tag{3.1}$$

which leads to the cigar geometry. Observables in the coset theory are obtained from observables in SL(2) $\times$ U(1) CFT by imposing invariance under (3.1) (and identifying gauge equivalent observables). Thus, one can use results about correlation functions in SL(2) CFT to study the coset.

Observables in SL(2) CFT[3] (see e.g. [20] for a recent discussion) are obtained by studying functions on the manifold H_3^+ and then applying to them the generators of the current algebra. A convenient basis for such functions is

$$\Phi_j(x,\bar{x}) = \frac{2j+1}{\pi}\left(\frac{1}{(\gamma-x)(\bar\gamma-\bar{x})e^{\phi}+e^{-\phi}}\right)^{2(j+1)}, \tag{3.2}$$

where $(\phi,\gamma,\bar\gamma)$ are coordinates on H_3^+ (the metric is $\frac{1}{k}ds^2 = d\phi^2 + e^{2\phi}d\gamma d\bar\gamma$) and $(x,\bar{x})$ are auxiliary variables introduced in [21]. In the quantum CFT on H_3^+, the Φ_j are primaries of the full SL(2) $\times$ SL(2) current algebra; their scaling dimensions are

$$\Delta_j = \bar\Delta_j = -\frac{j(j+1)}{k-2}. \tag{3.3}$$

For future reference we record their behavior for large ϕ (see [20] for a more detailed discussion),

$$\Phi_j \simeq e^{2j\phi}\delta^2(\gamma - x) + \cdots. \tag{3.4}$$

[2]The lower bound corresponds to pure $N=2$ Liouville or superconformal SL(2)/ U(1), where $c = 3+6/k = 15 \Rightarrow k = 1/2$.

[3]More precisely, we will be discussing the Euclidean analog of the SL(2) group manifold, $H_3^+ = \mathrm{SL}(2,\mathbb{C})/\,\mathrm{SU}(2)$.

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For studying the coset it is convenient to "Fourier transform" the fields $\Phi_j(x,\bar{x})$ and define the mode operators

$$\begin{aligned}\Phi_{j;m,\bar{m}} &= \int d^2x\, x^{j+m}\bar{x}^{j+\bar{m}}\Phi_j(x,\bar{x})\,,\\ \Phi_j(x,\bar{x}) &= \frac{1}{V}\sum_{m,\bar{m}}\Phi_{j;m,\bar{m}}x^{-m-j-1}\bar{x}^{-\bar{m}-j-1}\,,\end{aligned}\tag{3.5}$$

where $V \equiv \int d^2x/|x|^2$ is the volume of the boundary of H_3^+. The OPE algebra of the mode operators with the SL(2) currents is

$$\begin{aligned}J^3(z)\Phi_{j;m,\bar{m}}(w,\bar{w}) &\sim \frac{m\Phi_{j;m,\bar{m}}}{z-w}\,,\\ J^\pm(z)\Phi_{j;m,\bar{m}}(w,\bar{w}) &\sim \frac{(m\mp j)\Phi_{j;m\pm1,\bar{m}}}{z-w}\,,\end{aligned}\tag{3.6}$$

and similarly for $\bar{J}^A(\bar{z})$.

A set of observables in the coset theory is obtained by coupling $\Phi_{j;m,\bar{m}}$ to Y,

$$V_{j;m,\bar{m}} \equiv \Phi_{j;m,\bar{m}}e^{i\sqrt{2/k}(mY-\bar{m}\bar{Y})}\,.\tag{3.7}$$

Note that $V_{j;m,\bar{m}}$ is not charged under (3.1) and hence is physical. Its scaling dimensions are

$$\Delta_{j;m,\bar{m}} = -\frac{j(j+1)}{k-2}+\frac{m^2}{k}\,,\qquad \bar{\Delta}_{j;m,\bar{m}} = -\frac{j(j+1)}{k-2}+\frac{\bar{m}^2}{k}\,,\tag{3.8}$$

where $(m,\bar{m})$ run over a set of momenta and windings consistent with the fact that the radius of Y is $\sqrt{2k}$ [17]:

$$m = \frac{1}{2}(n_1+n_2k)\,,\qquad \bar{m} = -\frac{1}{2}(n_1-n_2k)\,.\tag{3.9}$$

One can think of n_1 as the momentum around the cigar (which is conserved) and n_2 as the winding (which is not).

Not all values of j are allowed in this theory. Non-normalizable vertex operators $V_{j;m,\bar{m}}$ correspond to real $j > -1/2$. Furthermore, unitarity implies that only operators with $j < (k-2)/2$ are kept [22]. Unitarity also implies that all scaling dimensions of Virasoro primaries in the CFT should be positive (except for the identity); in particular, (3.8) must satisfy $\Delta_{j;m,\bar{m}}, \bar{\Delta}_{j;m,\bar{m}} > 0$.

At large positive ϕ the form of the observables (3.7) simplifies. Using (3.4), (3.5) and (3.7) we find that

$$V_{j;m,\bar{m}} \sim e^{2j\phi}\gamma^{j+m}\bar{\gamma}^{j+\bar{m}}e^{i\sqrt{2/k}(mY-\bar{m}\bar{Y})}\,.\tag{3.10}$$

As explained in [23], in this region the theory simplifies and the target space can be thought of as a cylinder, $\mathbb{R}_\phi \times S^1$, labeled by ϕ and Y. The powers of γ, $\bar{\gamma}$ in (3.10) can be dropped and $V_{j;m,\bar{m}}$ go over to the standard observables in a linear dilaton vacuum.

We will be interested below in a supersymmetric version of the above coset CFT. This leads to a few small modifications in the analysis. If we denote by k the total level of the SL(2) current algebra on H_3^+, then the level of the bosonic part of the algebra is $k+2$ (with the fermions contributing -2 units). Thus, the scaling dimension of the primaries Φ_j (3.2) is shifted by $k \to k+2$ and the unitarity bound of [22] becomes

$$j < \frac{k}{2}\,. \tag{3.11}$$

Since the U(1) current J^3 that we are modding out by still has level k, formulae like (3.1) remain unchanged. Thus, in the superconformal coset SL(2)/ U(1) we have observables $V_{j;m,\bar{m}}$ given by (3.7) (with m, $\bar{m}$ satisfying (3.9)) whose scaling dimensions are

$$\Delta_{j;m,\bar{m}} = \frac{m^2 - j(j+1)}{k}\,, \qquad \bar{\Delta}_{j;m,\bar{m}} = \frac{\bar{m}^2 - j(j+1)}{k}\,. \tag{3.12}$$

Positivity of the scaling dimensions (3.12) implies in this case the constraint

$$m^2, \bar{m}^2 > j(j+1)\,. \tag{3.13}$$

Two and three point functions of the SL(2) primaries (3.2) in H_3^+ CFT were computed in [24]. The two point function is

$$\langle \Phi_{j_1}(x_1; z_1)\Phi_{j_2}(x_2; z_2)\rangle = C(j_1; k)|x_1 - x_2|^{-4(j_1+1)}|z_1 - z_2|^{-4\Delta_{j_1}}\delta(j_1 - j_2)\frac{\Gamma(1 - \frac{2j+1}{k})}{\Gamma(\frac{2j+1}{k})}\,, \tag{3.14}$$

where $C(j;k)$ is a numerical factor which can be found in [24]. Since we will be primarily interested in the analytic structure of the two point function, it is sufficient to note that $C(j;k)$ does not have poles or zeroes for finite j and $k > 1$. We have written (3.14) in a notation suited for application to the supersymmetric case, i.e. the level of the bosonic $\widehat{\mathrm{SL}(2)}$ implied in (3.14) is $k+2$. The two point function (3.14) does not have any singularities for all j satisfying the inequality

$$-\frac{1}{2} < j < \frac{k-1}{2}\,. \tag{3.15}$$

As j approaches the lower bound in (3.15), the two point function goes to zero, while when it approaches the upper bound it diverges.

Clearly, for physical applications (e.g. for string theory on AdS_3) both types of singularities are undesirable. For example, outside of the range (3.15) states in the spacetime CFT of string theory on AdS_3 have negative norm. Fortunately, the lower bound in (3.15) is precisely the condition for non-normalizability of the vertex operators discussed above, while the upper bound almost coincides with the unitarity constraint (3.11). In fact, (3.15) is slightly stronger. Note that this is not in contradiction with [22]. In these papers it is assumed that the current algebra primary state has positive norm, and one asks whether there are any negative norm states in the current algebra block obtained by acting on the primary with creation operators J^A_{-n} (after modding out U(1)). The two point function (3.14) instead probes the question whether the norm of the primaries changes sign as one varies j. Thus, below we will impose (3.15) on the observables.

Using (3.14) one can now compute the two point functions of observables in the coset theory (3.7). The first step is to transform from the "local fields" $\Phi_j(x,\bar{x})$ (3.2) to the modes (3.5). To simplify the formulae we will set $m=\bar{m}$, i.e. restrict to pure winding modes around the cigar (3.9). One finds

$$\begin{aligned}&\langle \Phi_{j_1;m,m}\Phi_{j_2;-m,-m}\rangle = \\ &\quad = \pi\delta(j_1-j_2)C(j_1;k)\frac{\Gamma(1-\frac{2j+1}{k})\Gamma(1+j+m)\Gamma(j-m+1)\Gamma(-2j-1)}{\Gamma(\frac{2j+1}{k})\Gamma(-j-m)\Gamma(m-j)\Gamma(2j+2)}\end{aligned} \tag{3.16}$$

(here and below we supress the z dependence).

To compute the two point function of coset observables $V_{j;m,m}$ (3.7) we multiply (3.16) by the two point function of the exponential in Y. This does not change the result; hence we find

$$\begin{aligned}&\langle V_{j_1;m,m}V_{j_2;-m,-m}\rangle = \langle \Phi_{j_1;m,m}\Phi_{j_2;-m,-m}\rangle = \\ &\quad = \pi\delta(j_1-j_2)C(j_1;k)\frac{\Gamma(1-\frac{2j+1}{k})\Gamma(1+j+m)\Gamma(j-m+1)\Gamma(-2j-1)}{\Gamma(\frac{2j+1}{k})\Gamma(-j-m)\Gamma(m-j)\Gamma(2j+2)}\,.\end{aligned} \tag{3.17}$$

3.2 Two point functions in double scaled LST

To use the results of the previous subsection for calculating two point functions in double scaled little string theory we have to construct BRST invariant observables in string theory on

$$\frac{\mathrm{SL}(2)_k}{\mathrm{U}(1)}\times\mathbb{R}^{d-1,1}\times LG(W=F)\,. \tag{3.18}$$

The analysis is very similar to that of [4, 16]. We will focus on (NS,NS) sector fields, since the rest of the observables can be obtained from these by applying the spacetime supercharges.

The lowest lying states are the "tachyons," described by the $(-1,-1)$ picture vertex operators

$$T(k_\mu)=e^{-\varphi-\bar{\varphi}}e^{ik_\mu x^\mu}V_{j;m,m}\,, \tag{3.19}$$

where k_μ ($\mu=0,1,\ldots,d-1$) is the momentum along $\mathbb{R}^{d-1,1}$ and φ, $\bar{\varphi}$ are the left and right moving bosonized superghost fields. The mass-shell condition and GSO projection lead to the following physical state constraints:

$$\frac{1}{2}k_\mu k^\mu+\frac{m^2-j(j+1)}{k}=\frac{1}{2}\,,\qquad m\in\frac{k}{2}(2\mathbb{Z}+1)\,. \tag{3.20}$$

Another set of observables corresponds to gravitons, whose $(-1,-1)$ picture vertex operators have the form

$$e^{-\varphi-\bar{\varphi}}\xi_{\mu\nu}\psi^{\mu}\bar{\psi}^{\nu}e^{ik_{\mu}x^{\mu}}V_{j;m,m}\,, \tag{3.21}$$

where ψ^{μ} are the worldsheet fermions on $\mathbb{R}^{d-1,1}$, $\xi_{\mu\nu}$ is the polarization tensor, and the physical state constraints are

$$\frac{1}{2}k_{\mu}k^{\mu}+\frac{m^2-j(j+1)}{k}=0\,,\qquad m\in k\mathbb{Z}\,. \tag{3.22}$$

There are also transversality conditions on $\xi_{\mu\nu}$ which we will not specify.

The most general (NS,NS) sector observable is a linear combination of vertex operators of the following form:

$$e^{-\varphi-\bar{\varphi}}P_{N_L,N_R}(x^{\mu},\psi^{\mu},\bar{\psi}^{\mu})\tilde{P}_{\tilde{N}_L,\tilde{N}_R}(\psi^{A},\bar{\psi}^{A},J^{A},\bar{J}^{A})V_{LG}e^{ik_{\mu}x^{\mu}}V_{j;m,m}\,, \tag{3.23}$$

where P_{N_L,N_R} is a polynomail in ∂x^{μ}, $\partial^2 x^{\mu}$, ..., $\bar{\partial}x^{\mu}$, $\bar{\partial}^2 x^{\mu}$, ..., ψ^{μ}, $\bar{\psi}^{\mu}$ and their derivatives; N_L and N_R are its total left and right-moving scaling dimensions, respectively. Similarly, $\tilde{P}_{\tilde{N}_L,\tilde{N}_R}$ is a polynomial in J^A, $\bar{J}^A$, their worldsheet superpartners ψ^A, $\bar{\psi}^A$ and their derivatives, with total scaling dimensions $(\tilde{N}_L,\tilde{N}_R)$. V_{LG} is a vertex operator from the $LG(W=F)$ sector of (3.18). BRST invariance and the GSO projection require (3.23) to be a bottom component of a worldsheet $N=1$ superfield, and in addition to satisfy

$$\frac{1}{2}k_{\mu}k^{\mu}+\frac{m^2-j(j+1)}{k}+N_L+\tilde{N}_L+\Delta_{LG}=\frac{1}{2}\,,\qquad Q_{LG}+\frac{2m}{k}+F_L\in 2\mathbb{Z}+1\,, \tag{3.24}$$

where Δ_{LG} and Q_{LG} are the left-moving scaling dimension and $\mathrm{U}(1)_R$ charge of V_{LG}, respectively, and F_L is the left-moving fermion number of (3.23) (similar relations hold for the right-movers). Moreover, the operator (3.23) must be invariant under the symmetry (3.1). Note also that $2m/k$ is not in general integer, unlike (3.9) with $m=\bar{m}$. The reason for that is that the GSO projection acts as an orbifold on $\mathrm{SL}(2)/\mathrm{U}(1)$ (see [4]), coupling it to the rest of the background (3.18).

We are now ready to compute correlation functions of the observables (3.19), (3.21) and (3.23). Recall that correlation functions of non-normalizable on-shell vertex operators in the bulk theory are interpreted in the boundary theory as off-shell Green's functions [7, 8]. Thus, their analytic structure provides information about the spectrum and interactions of the boundary theory. The analytic structure of the two point function, for example, contains information about the spectrum of the boundary theory. We will next study it in our case.

For the general vertex operator (3.23), the only non-trivial part of the two point function is the worldsheet correlation function $\langle V_{j;m,m}V_{j;-m,-m}\rangle$. This is the only source of singularities of the amplitude, and thus we will focus on it. As a first example, consider the two point function of the tachyon field (3.19),

$$\langle T(k_\mu)T(-k_\mu)\rangle = \pi C(j;k)\frac{\Gamma(1-\frac{2j+1}{k})\Gamma(1+j+m)\Gamma(j-m+1)\Gamma(-2j-1)}{\Gamma(\frac{2j+1}{k})\Gamma(-j-m)\Gamma(m-j)\Gamma(2j+2)}\,, \tag{3.25}$$

where j, m satisfy the constraints (3.13), (3.15), and (3.20). Note that the $\delta(j-j)$ from (3.17) has disappeared; it cancelled against a zero mode of the ghosts which normally would make the two point function vanish in string theory.

The two point function (3.25) has a series of poles, which we interpret as contributions of on-shell states in LST which are created from the vacuum by the operator $T(k_\mu)$. We next analyze these poles.

One series of poles corresponds to

$$m = j+n\,, \qquad n = 1,2,3,\ldots \tag{3.26}$$

It is easy to check that these are single poles. The residues of the poles all have the same sign, in agreement with the expected unitarity of LST. The corresponding masses of excitations are determined by the first line in (3.20):

$$M_n^2 = -k^\mu k_\mu = -1+\frac{2}{k}\left(j(2n-1)+n^2\right)\,. \tag{3.27}$$

For the smallest posssible value of m, $m = k/2$ (recall the second line in (3.20) and (3.26)), we find (using (3.15)) states with $n = 1,2,\ldots,[k/2]$, with masses

$$M_n^2 = \frac{2}{k}(n-1)(k-n)\,. \tag{3.28}$$

The lowest lying state is massless; it is followed by a finite series of excited states. More generally, we have

$$\begin{aligned} m &= \frac{k}{2}l\,, \qquad l = 1,3,5,7,\ldots \\ M_{n,l}^2 &= l-1+\frac{2}{k}(n-1)(kl-n)\,, \\ kl+1 &> 2n > k(l-1)+1\,. \end{aligned} \tag{3.29}$$

The range of n is obtained from (3.15). Note that Lorentz invariance of the LST implies that all the states that can be created from the vacuum by the tachyon field $T(k_\mu)$ (and whose masses are given by (3.29)) have spin zero.

The poles of (3.25) which are related to (3.26) by $m \to -m$ are not independent of those discussed above. All other poles violate one or more of the bounds[4] (3.13) and (3.15).

[4] Note, for example, that if we imposed the constraint (3.11) rather than (3.15), we would be lead to the rather unpleasant prediction that LST has tachyons. The pole of (3.25) at $m = k/2$, $j = (k-1)/2$ leads (3.20) to $M^2 = -1+1/2k$, which is in general negative. Furthermore, if k is integer (3.25) has a *double* pole at that value of j, which seems difficult to interpret in LST.

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The massless state corresponding to $n = l = 1$ in (3.29) has a natural interpretation in the fivebrane theory. We will describe it in the six dimensional maximally supersymmetric case, but the discussion can probably be extended to general singularities (1.6). As we saw in section 1, six dimensional double scaled type IIB LST describes the dynamics of $N = k$ fivebranes at the point (1.4) in the Coulomb branch. Due to the Higgs mechanism, the only massless states one expects to find in this theory are $N-1$ supermultiplets corresponding to the Cartan subalgebra of SU(N). The massless state with $n = 1$ in (3.28) belongs to one of these $N-1$ supermultiplets.[5]

It is not difficult to exhibit the other $N-2$ massless multiplets. For this, we need to examine the two point functions of tachyons (3.19) which have a non-trivial wave function in the minimal model denoted by $LG(W = F)$ in (3.18). In our case the superpotential is (1.1)

$$W = z_1^N + z_2^2 + z_3^2 \,, \tag{3.30}$$

corresponding to an $N = 2$ minimal model. The chiral worldsheet operators can be written as z_1^i, with $i = 0, 1, \ldots, N-2$. A natural generalization of the tachyon vertex (3.19) is

$$T_i(k_\mu) = e^{-\varphi-\bar{\varphi}} e^{ik_\mu x^\mu} z_1^i V_{j;m,m} \,, \tag{3.31}$$

where

$$\frac{1}{2}k_\mu k^\mu + \frac{m^2 - j(j+1)}{k} + \frac{i}{2N} = \frac{1}{2} \,, \qquad m = \frac{k}{2}l \,, \qquad l \in -\frac{i}{N} + 2\mathbb{Z} + 1 \,, \tag{3.32}$$

which is simply (3.24) for this particular case. The tachyon $T(k_\mu)$ (3.19) is in fact $T_0(k_\mu)$. The two point function of T_i is proportional to that of T_0, (3.25), and has poles giving rise to a spectrum similar to (3.29). It is easy to check that there is a physical pole at $m = kl/2$, $l = 1 - i/N$, $m = j+1$, which gives rise to a massless state in six dimensional LST. There are thus precisely $N-1$ massless states, corresponding to $i = 0, \ldots, N-2$, in agreement with the above expectations from SU(N) gauge theory.

Moving on to the two point function of the graviton, we see that in this case $m = kl/2$ with $l \in 2Z$. The two point function again has poles for j satisfying (3.26), which using (3.22) corresponds to the mass spectrum

$$\begin{gathered} M_{n,l}^2 = l + \frac{2}{k}(n-1)(kl-n) \,, \\ l = 2, 4, 6, \ldots \\ kl + 1 > 2n > k(l-1) + 1 \,. \end{gathered} \tag{3.33}$$

[5]It is sufficient to exhibit any one of the members of the supermultiplet, since the supersymmetry structure, worked out in [4], then guarantees the appearance of the rest of the supermultiplet, with the right quantum numbers.

All the states that the bulk graviton (3.21) couples to are massive.[6] This is consistent with the fact that the boundary theory is not a theory of gravity — the spectrum does not contain a massless spin two particle.

Consider, finally, the two point function of the most general on-shell vertex operator (3.23). Let $N \equiv N_L + \tilde{N}_L + \Delta_{LG}$ be the total excitation level corresponding to this vertex operator. As before, the poles correspond to (3.26), which in this case leads to the spectrum

$$\begin{aligned} M^2_{n,l} &= 2N + l - 1 + \frac{2}{k}(n-1)(kl-n)\,, \\ kl + 1 &> 2n > k(l-1) + 1\,, \end{aligned} \tag{3.34}$$

where l is defined as in the first line of (3.29), $m = kl/2$. It runs over the range implied by the second line in (3.24), $l \in -Q_{LG} - F_L + 2Z + 1$, subject to the unitarity constraint

$$\frac{m^2 - j(j+1)}{k} + \Delta_{LG} > 0\,, \tag{3.35}$$

generalizing (3.13) to operators which have non-trivial projection in $LG(W = F)$.

3.3 Three point functions

Three point couplings between the states (3.34) can be obtained from the three point function in the usual way (by taking the external legs on-shell and computing the residue of the resulting poles). To find the three point functions in the double scaled LST one follows the same steps as in the computation of the two point functions. We will only present the procedure schematically, leaving the details for future work. First, one needs the three point functions of the SL(2) primaries (3.2) in the H_3^+ CFT. Those are given in [24]:

$$\langle \Phi_{j_1}(x_1,\bar{x}_1)\Phi_{j_2}(x_2,\bar{x}_2)\Phi_{j_3}(x_3,\bar{x}_3)\rangle = D(j_1,j_2,j_3;k)C(j_1,j_2,j_3;x_1,x_2,x_3)\,, \tag{3.36}$$

where

$$C(j_1,j_2,j_3;x_1,x_2,x_3) = |x_1 - x_2|^{2(j_3-j_1-j_2-1)}|x_1 - x_3|^{2(j_2-j_1-j_3-1)}|x_2 - x_3|^{2(j_1-j_2-j_3-1)}\,. \tag{3.37}$$

$D(j_1,j_2,j_3;k)$ does not depend on x, and its explicit form is given in [24]. To compute the three point functions in the coset theory we need to transform the fields $\Phi_{j_i}(x_i,\bar{x}_i)$, $i = 1,2,3$, to the modes (3.5); this leads to integrals of hypergeometric functions depending on j_i, m_i. Finally, the three point functions in the spacetime theory are obtained by following the same steps as in subsection 3.2.

[6] The two point function of the graviton provides an example of the importance of the constraint (3.13). Ignoring it, one finds a series of poles of (3.17) at $m = 0$, $j = (n-1)/2$, $n = 2,4,6,\ldots$, which, if present, would correspond to tachyons in LST with masses which can be read off (3.22), $M^2 = -2j(j+1)/k$.

4. Discussion

By defining Little String Theory in a double scaling limit (see section 2), we were able to study the theory at weak coupling, $1/x$ (1.7). We used this description to compute correlation functions in the theory to leading order in $1/x$.

One of the main results of this paper is the calculation of the two point functions of observables in double scaled LST ((3.25) and its generalization to other vertex operators), and the corresponding spectrum of masses (3.34).

The two point functions we computed have a relatively simple analytic structure. They exhibit a series of single poles which we interpreted as arising from single particle states in LST, created from the vacuum by applying different operators.[7] This might come as a surprise, since in general one would expect two point functions in a non-trivial theory such as LST to be complicated functions of the momenta. The eigenstates of the hamiltonian need not be interpretable in terms of free particles, and one would expect a complicated structure containing resonances, thresholds (branch cuts) and "brown muck."

Of course, crucial to the simplicity of the two point functions is the fact that the theory is weakly coupled. As mentioned above, we only computed the two point function to leading order in $1/x$, and it is reasonable to expect the theory to be free in this limit. A good analogy is confining $\mathrm{SU}(N)$ gauge theory in four dimensions. For finite N the spectrum of the theory is complicated and, correspondingly, the two point functions of gauge invariant observables are expected to have a complicated analytic structure. As $N \to \infty$ the theory simplifies and becomes essentially a free field theory of the bound states (glueballs). The two point functions of observables like $\mathrm{Tr} F_{\mu\nu} F^{\mu\nu}$ are expected to simplify in the limit and have an analytic structure similar to the one found here.

Another interesting fact about the two point functions is that while the full spectrum of masses we find is rather rich, most operators do not couple to most of the states! There are two aspects to this, one that seems natural, and a second one that looks more surprising.

The natural one is the following. Even if the spectrum of LST at large x consists of free particles with the masses (3.34), one would in general expect that in the absence of special symmetries, generic observables can create from the vacuum both single and multi-particle states. The latter would contribute branch cuts in the two point functions. The fact that we only find poles means that the observables that we study, on-shell vertex operators in the bulk theory, only couple to single particle states in LST. This seems natural, since they are single string states in the bulk, and we are performing an expansion in g_s. In the gauge theory analogy mentioned

[7] As we pointed out, a nice consistency check is that the residues of all poles in the two point functions have the same sign, which is necessary for unitarity of LST.

above, fundamental string vertex operators are analogs of single trace operators in large N Yang-Mills theory.

More surprisingly, we find that even within the set of single particle operators/states, most operators do not couple to most states. The tachyon operators $T(k_\mu)$ (3.19) only couple to states (3.34) with $N = 0$, the gravitons (3.21) couple to states with $N = 1/2$, etc. This is not expected to happen in general. E.g. in the large N gauge theory example the spectrum is believed to exhibit Hagedorn growth, and one expects that all single trace operators have a non-zero overlap with all states which have the same quantum numbers (e.g. the same spin). For example, the operator $\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}$ mentioned above, should be able to create from the vacuum all states with spin zero.

The fact that in LST we find an infinite number of additional selection rules suggests that the weakly coupled theory has a very large symmetry group. It would be nice to understand it better.

It is natural to suggest that in LST there is a state-operator correspondence, namely, that each operator in the bulk can create a particular one particle state in the boundary theory. For example, a tachyon vertex operator (3.19) with given n, l (or j, m) can create from the vacuum a unique state, with mass given by (3.29). If this proposal is correct, it is easy to compute the degeneracy of states with any mass (3.34). Since the mass spectrum (3.34) is very similar to that of the bulk theory, and the state-operator correspondence suggests that the degeneracies are the same as well, one finds a Hagedorn density of states

$$\rho(E) \sim e^{\beta_H E}\,, \tag{4.1}$$

where β_H, the inverse Hagedorn temperature, is the same as that of the bulk theory. We emphasize that this last conclusion assumes the state-operator correspondence, and should be examined more carefully.

Note that all the poles of the two point functions that give rise to the masses (3.34) occur for values of (j, m) that satisfy (3.26) (or, equivalently, its mirror image related by $m \to -m$). Algebraically, these are values for which the underlying SL(2) representations are one sided. They contain a state, $m = j + 1$, which is annihilated by J^- (or $m = -(j + 1)$, annihilated by J^+). These representations are sometimes refered to as the principal discrete series. A natural question is why does the principal discrete series play a special role in the analysis, and how one can think of the resulting spectrum of the LST.

We would like to make two comments regarding these issues. The first is that the appearance of the principal discrete series is not particularly surprising if we compare to the better understood case of string theory on AdS_3 [7, 25]. In that case, the holographically dual spacetime theory is a two dimensional CFT and the operators $\Phi_j(x, \bar{x})$ (3.2) correspond to local operators in this theory.

While there are well known subtleties [26, 24] with the state-operator correspondence in CFT on AdS_3, the *spacetime* CFT is a more or less conventional unitary CFT, and in particular it does have a state-operator correspondence. The state corresponding to a given operator $V\Phi_j(x,\bar{x})$ (V is a vertex operator which depends on the rest of the string background; see [25] for the details) is obtained by acting with $V\Phi_j(x,\bar{x})$ on the vacuum. In radial quantization the 'in' vacuum corresponds to $x = 0$, hence incoming states correspond to $\lim_{x\to 0} V\Phi_j(x,\bar{x})|0\rangle$.

Expanding $\Phi_j(x,\bar{x})$ in modes, as in (3.5), we see that the mode that creates the above state in the spacetime CFT has $m = \bar{m} = -j-1$. Modes with larger m must kill the vacuum, while those with smaller m do not contribute in the limit $x \to 0$. Similarly, there are states created by acting on the vacuum with $\partial_x^{n-1}\partial_{\bar{x}}^{n-1}\Phi_j(x,\bar{x})$. The modes that create these states have $m = -j-n$. The modes (3.26) discussed in section 3 are related to them by $m \to -m$; they create states when acting on the 'out' vacuum at $x \to \infty$.

Thus, at least in string theory on AdS_3 the modes (3.26) corresponding to the principal discrete series certainly do create physical states in the spacetime theory. It is perhaps not surprising that something very similar seems to be going on in the SL(2)/ U(1) case.

The second comment concerns the physical interpretation of the spectrum (3.34). It is known [27] that the dynamics of winding modes on the cigar SL(2)/ U(1) is equivalent by T-duality to the motion of a particle in a potential which is attractive for small ϕ and goes to a constant at large ϕ. The spectrum contains bound states corresponding to the principal discrete series, and a continuum above a gap, which corresponds to the normalizable states with $j = -1/ + i\lambda$. The discrete states can presumably be thought of in the original variables as some sort of bound states whose wave function is supported near the tip of the cigar. The states with $j = -1/2 + i\lambda$ make wave packets that can live arbitrarily far from the tip of the cigar and have an arbitrary momentum λ (hence the continuum). It is easy to verify using (3.24), (3.34) that the continuum starts right above the highest mass discrete state (in a sector with given N, l), as expected from the form of the potential for the winding modes [27].

One can ask whether the normalizable states with $j = -1/ + i\lambda$ should be included in the LST. For this to be the case, it seems to us necessary that there are *some* observables in the theory which can create these states from the vacuum. The class of observables analyzed in this paper, the non-normalizable fundamental string operators, does not seem to couple to these states. If no other observables do either, we would be inclined to believe that these modes decouple from the LST dynamics.[8]

[8]This issue is further confused by the fact [24, 26] that in correlation functions of non-normalizable operators in CFT on SL(2)/ U(1) one is typically instructed to sum over all the normalizable states in internal channels. Nevertheless, it seems that the singularity structure of the resulting correlation functions can be interpreted purely in terms of states bound to the tip of the cigar.

Returning to the spectrum (4.1), our conclusion that the density of states of the LST grows exponentially with energy, with the same Hagedorn temperature as that of the bulk theory, seems to be at odds with the discussion of [28] where it was argued that the Hagedorn temperature is in fact lower by (roughly) a factor of $\sqrt{k}$ (and thus the density of states ρ grows more rapidly with energy than (4.1)).

Our attitude to this apparent discrepancy is the following. The analysis of [28] applies in our language to energies $E \gg m_W \sim x$, while we computed the spectrum of excitations with energies $E \ll m_W$. We find a Hagedorn growth of the density of states (4.1) with a smaller β_H than that of [28] in the range $m_s \ll E \ll m_W$. It is reasonable to expect that the density of states with $E \gg m_W$ is much higher and agrees with [28].

The regime $E \gg m_W$ is strongly coupled in our description and is difficult to study directly. As has been learned in many examples in the last few years, typically only states that are protected by supersymmetry can be reliably counted at strong coupling. Perhaps D-brane states which preserve some of the supersymmetry in the background (3.18) would resolve the discrepancy (see [29] for a recent discussion).

Finally, some of the vacua of LST that our discussion applies to correspond to theories on fivebranes wrapped around Riemann surfaces, which are relevant for describing strongly coupled gauge theories using branes [4]. We hope that our results will help improve this understanding.

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